Market Microstructure and Informational Efficiency

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The competitive market is informationally efficient; people only need to know prices to implement a competitive outcome. However, the standard formulation of competition neglects any underlying market microstructure; prices—which provide all necessary information—are exogenous. This paper studies the informational efficiency of two market microstructures: search versus market-makers. First, we prove that a large search economy requires an infinitely larger message space compared to the competitive equilibrium. Then, we prove that an economy with market-makers only needs one more dimension than the competitive process, making it is second-best in terms of information but with an explicit model of decentralized price-formation.

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1 Introduction

Economists use two swords to defend competitive markets. First, competitive markets exhaust all gains from trade; that is, competitive markets are Pareto efficient (First Welfare Theorem). Second—and the focus of this paper—prices in competitive markets can communicate all the relevant information dispersed throughout the economy; that is, competitive markets are informationally efficient (Hayek Hypothesis). To use the fa-
mous example from Hayek (1945, p. 526), when the price of tin increases, "All that the users of tin need to know is that some of the tin they used to consume is now more profitably employed elsewhere." Knowing whether demand or supply shifted to cause the price increase would be redundant information, as far as the tin user is concerned. In a competitive market, decision-makers only need to consider minimal information for their decision problems and competition still achieves an efficient allocation of resources.

The Hayek Hypothesis was formalized in a series of landmark papers, such as Mount and Reiter (1974), Hurwicz (1977a, 1977b, 1977c), Jordan (1982), and Chander (1983), which developed a formal concept of informational efficiency as a minimal message space. Mount and Reiter (1974) showed that competitive equilibria are informationally efficient in the sense that competitive prices communicate the minimum amount of information necessary to implement a Pareto efficient allocation in an environment where information is dispersed. Jordan (1982) proved that competitive prices are the unique decentralized mechanism that achieves informational efficiency and satisfies the individual rationality constraint.

While these formal informational efficiency results are important, the process that determines market prices is not modeled; only the quantities are determined by the decentralized choices of decision-makers while the prices are determined by the "Walrasian auctioneer" or "the economist", who solves the model by equating the quantity supplied with the quantity demanded. Without any way to measure the information required to reach the competitive allocation through a fully decentralized strategic procedure, it is unclear how seriously economists should take these informational efficiency results; the results are about the informational efficiency of prices in a model where price formation is not modeled.

The purpose of this paper is to study the information efficiency in markets with strategic price-setters and without the Walrasian auctioneer. Therefore, we need to explicitly incorporate the market microstructure; that is, we need to model any intermediators and

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2. Dispersed means that each individual has private information regarding her own endowments, preferences, or technology.

3. These results were extended to convex economies with public goods and externalities (Sato 1981; Tian 2004). More recently, Nisan and Segal (2006) extended this literature to non-convex economies and the analysis of the allocation for indivisible goods.
other institutions that facilitate trade (Spulber 1996b, p. 135). Following the market microstructure literature, such as Gehrig (1993), Spulber (1996a, 2002), and Rust and Hall (2003), we compare two forms of trade we observe in the world: direct trade between buyers and sellers and indirect trade through market-makers. The original informational efficiency literature or more modern mechanism design papers, such as Mookherjee and Tsumagari (2014), take an alternative approach and study communication costs of general, abstract allocation mechanisms.

To isolate the informational efficiency role of the market-makers, we focus on the relevant frictionless limit, where the allocation approximates the competitive equilibrium. At the frictionless limit, the only difference between the search, market-marker, and competitive markets is in how the prices emerge: bargaining by searchers, set by market-makers, or set by the auctioneer. Therefore we know we are not implicitly bringing in any additional friction for the market-makers to resolve.\footnote{Spulber (1996b) places price setting as the first role of market-makers, above other roles such as searching or monitoring.}

Our paper has two main results. First, we show that a standard search model, based on Mortensen and Wright (2002), is unattractive from an informational efficiency perspective. Even as search frictions approach zero so that the mechanism is approximately competitive and therefore approximately efficient from an allocative perspective, search is incredibly inefficient from an informational perspective. Formally, we prove that, for large economies, the search allocation mechanism requires infinitely more information than the competitive mechanism. The reason for this is that, in a search economy, each buyer needs to know the price he or she would trade with each seller. For example, if there are $k$ buyers and $k$ sellers who trade one good, the message space must include $k^2$ prices. Therefore, the size of the information grows at $O(k^2)$. Due to the informational costs, and not just shoe-leather costs, search may be too costly in large economies.

Second, we show how an alternative market microstructure with market-makers can approximate the informational efficiency of the competitive mechanism. If information is costly to process, agents may want to rely on intermediators to do that work. Following the intermediation models of Gehrig (1993), Spulber (1996a, 1996b), and Rust and Hall (2003), we introduce a model with market-makers who intermediate trade between buyers and sellers in a model of dynamic price formation.\footnote{Following Rust and Hall (2003), we use the term market-maker because they operate an exchange. There are many subtleties of the market microstructure that we do not study. See Spulber (2019) for a recent discussion.} We show that a model of ex-
change through intermediators can approximate the informational efficiency properties of the competitive equilibrium. Because most agents are not market-makers, buyers and sellers can act as they do in the competitive mechanism, and the size of the information only grows at \( O(k) \), which is the same order as the competitive model. However, unlike the competitive model, the market-maker model endogenizes the equilibrium prices as strategic choices of decision-makers.

Market-makers allow other agents to economize on information through an "intellectual division of labor." Unlike in the search economy, buyers and sellers "outsource" the price formation process to intermediators. After all, setting prices can be costly, so not everyone should do it. With positive trade frictions, the market-makers make profits by extracting part of the surplus of the gains from trade. These market-makers can be thought of as arbitrageurs who can buy low and sell high, exploiting opportunities that other actors are unaware of. As trade frictions go to zero, the market-makers still economize on information for the other agents, but the arbitrage opportunities disappear, and market-makers no longer make any profits and the allocation approximates the competitive allocation.

We then extend the general market-maker model to a dynamic setting with a monopoly market-maker. Other potential market-makers can pay a fixed cost to enter, so that the market is contestable (Baumol 1982). In equilibrium, the monopoly incumbent will deter entry and set all prices, much like the Walrasian auctioneer. Unlike the auctioneer, the market-maker is strategic, and prices are endogenous. The downside is that with positive frictions there are different bid and ask prices for the good, instead of the single price from the auctioneer. This means that to implement the market-maker allocation, the message space requires only one more dimension than the uniquely informationally efficient competitive market, making an economy with a monopoly market-maker second-best compared to the competitive market.

Our model market-maker model allows us to better understand the informational efficiency of "platforms", which are just another term for market-makers (Spulber 2019), and their growing role within the modern economy. Our results suggest that the presence of platforms with large market-shares, such as bitcoin exchanges, Amazon, and

6. Jordan (1982) proved that the competitive market is the unique informationally efficient mechanism, so the market-makers do require strictly more information than the competitive mechanism.
7. There is a branch of the literature, following Kirzner (1973), that calls these market-makers "entrepreneurs" who are "alert" to profit opportunities that exist in the market at a specific point in time.
8. The formal connection between no-arbitrage and competitive equilibrium is well understood in the case of product markets (e.g., Makowski and Ostroy 1998) and financial markets (e.g., Werner 1987).
Uber, might economize on information (in addition to any other frictions they reduce),
compared to industries with many agents on both sides of the market. As the com-
plexity of economic systems increases—for example, because of globalization—market-
maker/platforms have an increasing informational advantage over direct exchange. All
else being equal, we should predict an increase in the use of these platforms with eco-

Throughout the paper, we focus on informational efficiency in the sense of the size of
the message space. The message space can be interpreted as a measure of the complexity
of a verification protocol. As Segal (2010a, p. 228) explains

][I]magine an omniscient oracle who knows the agents’ valuations and conse-

quently the optimal allocation(s), but needs to prove to an ignorant outsider

that an allocation \( x \) is [a solution]. The oracle does this by publicly announcing

a message \( m \in M \). Each agent \( i \) either accepts or rejects the message, doing this

on the basis of his own type... The acceptance of message \( m \) by all agents must

verify to the outsider that allocation \( x \) is optimal... The (worst-case) complex-

ity of a verification protocol with message space \( M \) is the minimum number

of bits needed to encode a message, which is \( \log_2 M \).

For example, the original papers on informational efficiency focused on a Walrasian equi-

librium, which can be interpreted through the lens of a verification protocol. As Segal

(2010a, p. 229) further explains "The role of the oracle is played by the ‘Walrasian auc-
tioneer,’ who announces the equilibrium prices and allocation. Each agent accepts the

announcement if and only if his announced allocation constitutes his optimal choice from

the budget set delineated by the announced prices.” Thus, the agents verify the equilib-

rium.

Informational efficiency as the minimal message space should not be confused with

informational efficiency in the sense of "information aggregation," as used in papers such


notions share the same name and trace back to ideas in Hayek (1945), they are formally

distinct. We do not address information aggregation in this paper.
Informational Requirements and Foundations for Competitive Equilibrium

There is a large literature on strategic foundations for competitive equilibrium, which attempts to justify the assumption that markets can be described by the model of competitive equilibrium. This literature shows that under a variety of different conditions that the strategic equilibrium of decentralized economies (modeled such as the core of a cooperative game or as a matching and bargaining game) generates the same allocation as the competitive equilibrium. Therefore, the competitive equilibrium can be thought of as a convenient shortcut, instead of needing to model the more complicated, decentralized process.

Unlike with competitive markets, the search literature has mostly assumed that decision-makers have complete information regarding market conditions. More recent papers have incorporated slightly less than complete information, such as asymmetric information regarding private valuations (Satterthwaite and Shneyerov 2007, 2008), private information regarding asset values (Golosov, Lorenzoni, and Tsyvinski 2014; Babus and Kondor 2018), and imperfect information regarding aggregate market conditions (Laermann, Merzyn, and Virág 2018). However, these studies assume that decision-makers have full awareness of the data of the economy: there is only uncertainty regarding the "quality" of the information. There is no constraint on the "quantity" of information regarding the economy that can be utilized by each decision-maker, as in papers on communication and informational costs.9

In contrast to the (almost) complete information in search models, the literature on informational efficiency argues that, in a competitive equilibrium, individual decision-makers only need to be aware of the minimum amount of parameters about the state of the market to achieve an efficient allocation. The decision-makers could simply be completely unaware of the rest of the economy besides market prices. By unawareness, we mean not "generally taking into account", "being present in mind" (Modica and Rustichini 1999, p. 274), "thinking about" (Dekel, Lipman, and Rustichini 1998), or "paying attention to" (Schipper 2014).

Despite claims that efficient competitive markets require complete information,10 the formalizations of the Hayek Hypothesis prove that, even when agents are unaware of

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9. For example, see Segal (2007, 2010b), Fadel and Segal (2009), Mookherjee and Tsumagari (2014), and Ashlagi et al. (2020).
10. This was even the conventional wisdom into the 1960s (Smith 2015, p. 242).
most of the economy, competitive markets achieve the same allocation as a central planner in possession of all the data in the economy who is maximizing a social welfare function. As classic studies like Smith (1982) show through experiments, with proper trading rules, competitive outcomes emerge even with "strict privacy wherein each buyer in a market knows only his/her own valuation of units of a commodity, and each seller knows only his/her own cost of the units that might be sold." Unfortunately, on the theory side, the process through which a competitive market achieves equilibrium, specifically how prices emerge, is notoriously not modeled.

This is why it is helpful to study different market microstructures; they include different mechanisms for price formation. But do different market microstructures still retain the informational efficiency of competitive markets? This paper shows that it depends. If the allocation corresponding to a competitive equilibrium is reached through a distinct mechanism, then this informational efficiency property may not hold. Specifically, we show that information must grow at $O(k^2)$ in a typical random search model, making search infinitely less informationally efficient than the competitive mechanism for a large economy. We show that an economy with market-makers (possibly just one) who facilitate trade between buyers and sellers can require little information.

One indirect implication of our paper is to show that the degree of informational efficiency of the allocation mechanism that is implicit in the model should also be considered when judging the model. For example, one justification for using competitive models is that people need such little information to implement a competitive equilibrium.\textsuperscript{11} Models that explain price formation should also explain how the message space of the allocation mechanism that is implicit in the model approximates the main feature of the competitive mechanism: that agents can take terms of trade as a given without the need to "think" about how they are determined.

The rest of the paper is structured as follows: Section 2 lays out the abstract environment within which we will be considering the specific mechanisms. Section 3 explains the baseline competitive mechanism. Section 4 describes the search mechanism and includes our result on informational inefficiency. We then construct our market-maker in Section 5. Section 6 develops a dynamic model with a monopoly market-maker, and Section 7 concludes. All proofs are given in the Appendix.

\textsuperscript{11} The model needs to also fit the data, which, for the competitive model, is most clearly seen from experimental data (Smith 1982; Friedman 1984; Friedman and Ostrov 1995; Martinelli, Wang, and Zheng 2019)
2 Physical Environment

Following previous research on informational efficiency, we start talking about abstract environments and mechanisms. We then embed three specific environments and mechanisms (competitive, search, and market-maker) within that overall structure.

Consider a class of environments $E$ where there are two goods: a consumption good and a numeraire good, consumption set is $X = \mathbb{R}_+ \times \mathbb{R}$. There is a continuum of traders $I$, indexed by $i$, with their measure normalized to one. Each trader’s type specifies whether she is a buyer or seller and a number. If she is a buyer she has unit demand for the consumption good and the number specifies her valuation of the consumption good in terms of the numeraire. If she is a seller, she has a unit supply of the consumption good and the number specifies the opportunity cost to supply a unit of the good in terms of the numeraire. Let $\omega(i) \in \mathbb{R}^2_+$ be the endowment of trader $i$ (thus $\omega_1(i) = 0$ if $i$ is a seller and $\omega_1(i) = 1$ if $i$ is a buyer). Let $E_i \subset \{\text{Buyer, Seller}\} \times \mathbb{R}_+$ be the set of possible types for trader $i$. While we work with the continuum for mathematical simplicity, we will interpret the number of traders as the number of different types (each having equal measure). We will consider different cases in which the sets of values are finite and infinite. A specific environment is a realization of $e \in E$, where $e = (e^i)_{i \in [0,1]}$.

Let $Y$ be the set of feasible net trades. It is the set of measurable functions $y : I \rightarrow \mathbb{R}^2$ such that $y(i) + \omega(i) \in X$, and $\int_0^1 y_2(i) = 0$. The last condition means that net trades must add up to zero. Formally, we can define the set of feasible net trades as

$$Y = \left\{ y : I \rightarrow \mathbb{R}^2 : y(i) + \omega(i) \in X, \forall i \in I, \int_0^1 (y_1(i), y_2(i)) di = (0,0) \right\},$$

To formalize the notion of the informational size, we need to define messages. The set $M$ is an abstract message space. The non-empty valued correspondence $\mu : E \Rightarrow M$ specifies the messages for each environment. Finally, the outcome function $g : M \rightarrow Y$ maps messages to net trades.

Putting this together, we can define an allocation mechanism, which is the object of interest.

**Definition 1.** An allocation mechanism is a triple $(\mu, M, g)$.

We are interested in informationally decentralized allocation mechanisms, which are mechanisms that feature a message process $(\mu, M)$ that is privacy-preserving.
Definition 2. A message process \((\mu, M)\) is privacy-preserving if there exists a correspondence \(\mu^i : E^i \Rightarrow M\) for each \(i\) such that for each \(e \in E\) \(\mu(e) = \bigcap_{i \in [0,1]} \mu^i(e^i)\).

Because we will be focused on privacy-preserving mechanisms that do not depend on a trader’s index, we can simplify the description of the environment by rearranging traders into two sets. First, let \(s \in [0,1]\) denote the measure of sellers. Then, we can denote the set of sellers \(S = [0, s]\) and the set of buyers by \(B = [0, 1 - s]\), where \(b = 1 - s > 0\) is the measure of buyers). Finally, we can simplify the descriptions of valuations and costs through the cumulative distribution functions \(F\) and \(G\) for buyers and sellers. In general, \(F\) and \(G\) will not be continuous or strictly increasing. To allow the possibility for mutually beneficial trade, we will assume that the maximum value is greater than the minimum cost.

Approximating a Continuous Environment

While we always work with a continuum of traders, even when there are finite types, we will also want to consider a limit case where there are infinite types. We will call an environment continuous if \(F\) and \(G\) are continuous and strictly increasing over some interval: \([\underline{v}, \overline{v}] \subset \mathbb{R}_{++}, [\underline{c}, \overline{c}] \subset \mathbb{R}_{++}\). To allow for the possibility of mutually beneficial trade, we will assume \(\overline{v} > \underline{c}\).

With a continuum of types, the allocation \(y \in Y\) is an infinite-dimensional object. However, the arguments about the size of information messages in papers like Hurwicz (1977b) and Jordan (1982) use finite-dimensional messages. To make a direct comparison, we study sequences of environments with finitely many types of buyers and sellers, and as the number of types grows to infinity, the distributions of types approximate a continuous distribution of buyers \(F\) and sellers \(G\).

Let \(\{e^k\}_{k \geq 2}\) be a sequence of environments where buyer and seller types are distributed according to \(\{F^k, G^k\}_{k \geq 2}\), which are sequences of step-functions. A pair \((F^k, G^k)\) represents the cumulative distributions of types of buyers and sellers, respectively, in an environment \(e^k\) where there are \(k\) types of buyers, \(k\) types of sellers, and each type has measure \(1/k\). That is, the sets of buyers and sellers are partitioned into subsets of the same measure whose elements are all identical. The pair of sequences \(\{F^k, G^k\}_k\) converges to \(F\) and \(G\), respectively. We call an environment with \(k\) types of buyers and sellers a \(k\)-environment. By slight abuse of notation, \(I\) denotes the set of trader types in a \(k\)-environment.
3 Competitive Mechanism

First, consider an environment where $F$ and $G$ are continuous. Let $M_c = \{(p, y) \in \mathbb{R}_{++} \times Y : py_1(i) + y_2(i) = 0, \forall i\}$. For an environment $e \in E$, for each $i \in S \cup B$, define the correspondence $\mu^i_c : E \Rightarrow M_c$ by

$$
\mu^i_c(e^i) = \begin{cases}
(p, y) \in M_c : y(i) = \\
(1, -p) & \text{if } i \in B \text{ and } v^i \geq p \\
(0, 0) & \text{if } i \in B \text{ and } v^i < p \text{ or } i \in S \text{ and } c^i > p \\
(-1, p) & \text{if } i \in S \text{ and } c^i < p
\end{cases}.
$$

In words, a buyer purchases a unit of the good for $p$ if his valuation is higher, and a seller sells the good for $p$ is her cost is lower. Define the messages for the competitive environment as

$$
\mu_c(e) = \bigcap_{i \in [0,1]} \mu^i_c(e^i),
$$

which is privacy-preserving by construction. Then, $(\mu_c, M_c)$ is the competitive message process and $(\mu_c, M_c, g_c)$ is the competitive allocation mechanism, where $g_c : M_c \rightarrow Y$ is the outcome function given by $g_c(p, y) = y$.

**Remark 1.** Note that a competitive equilibrium price $p^*$ must satisfy feasibility, $sG(p^*) = b[1 - F(p^*)]$. If $G$ and $F$ are continuous and strictly increasing, the competitive equilibrium price is unique. This result is standard and is shown in Appendix Subsection A.1.

Now consider a $k$-environment, which by definition is not continuous and has finite types. The allocation mechanisms can be written in terms of those types. Let $\phi \in \{1, \ldots, k\}$ index buyer types and $\gamma \in \{1, \ldots, k\}$ seller types, and let $x(\phi)$ be the valuation of a buyer of type $\phi$ and $z(\gamma)$ be the cost of a seller of type $\gamma$.

Let $y^B(\phi) = (y^B(\phi)_1, y^B(\phi)_2)$ be the net trades for a buyer of type $\phi$ in the competitive equilibrium and $y^S(\gamma) = (y^S(\gamma)_1, y^S(\gamma)_2)$ the net trades for a seller of type $\gamma$. A profile of net trades specifies a net trade for each of the $2k$-types of traders: $y = \left((y^B(\phi))^{k}_{\phi=1}, (y^S(\gamma))^{k}_{\gamma=1}\right)$. The set of net trades is then

$$
Y^k_c = \left\{ y : I \rightarrow \mathbb{R}^2 : y(i) + \omega(i) \in X, \forall i \in I, \sum_{\phi=1}^{k} y^B(\phi) + \sum_{\gamma=1}^{k} y^S(\gamma) = (0, 0) \right\},
$$

as there are $k$ types of buyers and $k$ types of sellers.
The competitive message space in the $k$-environment specifies a price of $p$ and allocation $y$ where $y \in Y^k_c$. Let $(\mu^k_c, M^k_c, g^k_c)$ be the $k$-environment versions of the competitive allocation mechanisms, where

$$M^k_c = \{(p, y) \in \mathbb{R}_{++} \times Y^k_c : py^j(i) + y^j(i) = 0, \forall j \in \{B, S\}, \forall i \in \{1, \ldots, k\}\}.$$ 

Let $\mu^k_c$ be the finite analogue of $\mu_c$, which maps $E^k$ into $M^k_c$, and $g^k_c$ is the projection from $M^k_c$ to $Y^k_c$. In addition $F^k, G^k$ are such that $p^*$ is consistent with competitive equilibrium in economy $e^k$.

In the competitive mechanism of the $k$-economy, the message space includes only one price (as the price of the numeraire good is normalized to 1) and $2k$ types of buyers and sellers. However, if we know the trades for $2k - 1$ of the types, then market-clearing implies the trades for the last type. Therefore, we have the following lemma:

**Lemma 1.** The message space of the competitive mechanism $M^k_c$ is a $2k$-dimensional manifold.

**Proof.** See Appendix Subsection A.2

Jordan (1982) proved that the competitive mechanism is the unique informationally efficient mechanism. Thus, the competitive mechanism will serve as the benchmark that we will compare other mechanisms against.

## 4 The Informational Inefficiency of the Search Mechanism

### 4.1 Environment

The competitive mechanism is meant to capture a limit of some decentralized process, such as search and bargaining. To understand the informational efficiency of that decentralized, search process, we need to develop an explicit model. To keep the model standard, we use a baseline model from Mortensen and Wright (2002).

There is an entry rate of potential $s > 0$ sellers and $b = 1 - s > 0$ buyers. Given stocks of $B$ buyers and $S$ sellers currently in the market, buyers and sellers meet at the rate $M(B, S)$. Let the buyer/seller ratio $\theta = B/S$ be the market tightness parameter, $m(\theta) = M(B, S)/S$ be the rate a seller meets buyers, and $m(\theta)/\theta$ be the rate a buyer meets sellers. There are discount rates and search costs for buyers and sellers, given by $(r, e_b, c_s) \in \mathbb{R}^3_+$. For this search environment to be directly comparable to the competitive
mechanism, the flows of buyers and sellers exiting the market should be equal to the flows entering—\( b \) and \( s \)—so that the corresponding net trades in the competitive equilibrium have an analogous implementation in this environment. The search equilibrium where the rate of entry of new buyers and sellers in the market is the same as the exit rate is called a steady-state search equilibrium.

When a buyer and a seller meet, one of the two, randomly chosen, announces a take-it-or-leave-it price offer. Let \( \omega \in (0,1) \) be the probability a seller makes the offer. If the other party rejects the offer, they both continue searching as if they had never met; if the other party accepts the offer, the exchange occurs, and both exit the market.

Mortensen and Wright (2002) show that this bargaining protocol is equivalent to the generalized Nash solution over the joint surplus where the sellers’ bargaining power is \( \omega \in (0,1) \). To see this, let \( V_b(x) \) be the value of a buyer with valuation \( x \) to participate in the market and \( V_s(z) \) be the value of a seller with cost \( y \). A take-it-or-leave-it offer to a buyer of one unit of the good for a price \( p \) is acceptable if and only if the price generates a surplus equal or higher than the value of continuing to search for trading partners; thus, the offer, \( x - p \geq V_b(x) \), is acceptable to a seller if and only if \( p - z \geq V_s(z) \). Thus, given complete information, the best strategy for one party is to offer the other party’s reservation value; thus, the seller offers \( p = x - V_b(x) \), the buyer offers \( p = y + V_s(z) \), and a transaction occurs if and only if \( x - V_b(x) \geq y + V_s(z) \). Since the seller makes the offer with probability \( \omega \), the expected price of a transaction is

\[
p(x, z) = y + V_s(z) + \omega [x - z - V_b(x) - V_s(z)].
\]  

This is the price according to the generalized Nash solution if the seller captures a fraction \( \omega \) of the joint surplus given reservation values \( y + V_s(z) \) for the seller and \( x - V_b(x) \) for the buyer.

Given the transaction prices, the values of participating in the market can be described as follows: The expected value of participation in the market for a buyer satisfies

\[
rV_b(x) = \frac{m(\theta)}{\theta} \int \max\{x - p(x, z) - V_b(x), 0\}d\Gamma(z) - e_b,
\]  

and the expected value of participation in the market for a seller satisfies

\[
rV_s(z) = m(\theta) \int \max\{p(x, z) - z - V_s(z), 0\}d\Phi(x) - e_s,
\]
where $\Gamma$ and $\Phi$ are the distributions of seller and buyer types participating in the market. These distributions differ from the respective exogenous distribution of potential seller and buyer entrants, $G$ and $F$, as some types choose to not enter if the expected value of entering is not positive.

Substituting the right-hand side of equation 1 into equations 2 and 3 yields

$$rV_b(x) + e_b = \frac{m(\theta)(1 - \omega)}{\theta} \int \max\{x - z - V_b(x) - V_s(z), 0\}d\Gamma(z)$$

and

$$rV_s(z) + c_s = m(\theta)\omega \int \max\{x - z - V_b(x) - V_s(z), 0\}d\Phi(x).$$

Equations 4 and 5 show that the value of participating in the market is strictly increasing in the buyer’s valuations and strictly decreasing in the seller’s cost. Because the participation values are monotonic, there exist marginal entrants. The steady-state search equilibrium is defined in terms of a pair of marginal types of buyers and sellers $(R_b, R_s)$, where a buyer with valuation $x$ only enters the market if $x > R_b$, and the seller with cost $z$ only enters if $z < R_s$. In a steady-state search equilibrium, the distribution of types participating in the market is stationary, which implies that: (1) The measure of entering sellers and buyers must be the same, and therefore the pair of marginal valuations $(R_b, R_s)$ satisfies the condition $sG(R_s) = b[1 - F(R_b)]$. (2) The distribution of participating types is constant.

The steady-state search equilibrium is characterized by $(V_b, V_s, R_b, R_s, \Phi, \Gamma)$, the value functions $(V_b, V_s)$, cutoff valuations to participate in the market $(R_b, R_s)$, and the distributions of participating types $(\Phi, \Gamma)$ of buyers and sellers, respectively.

### 4.2 Characterization of the Search Equilibrium

To directly compare to the competitive equilibrium, we focus on search equilibria that approximate the frictionless limit when search costs converge to zero (where the Law of One Price holds). To see this, notice that as the discount rate $r$ goes to zero, the left-hand side of 4 does not depend on the buyer’s valuation $x$. Thus, the variation of the left-hand side with regards to $x$ converges to zero as $r \to 0$, which implies that $x - V_b(x)$ converges to a constant as $r \to 0$. In particular, for the marginal buyer type $R_b$ we know that $V(R_b) = 0$; thus, this constant is $R_b$. Therefore, $x - V_b(x) = R_b$ for $x \geq R_b$ when $r = 0$. Analogously for the seller case, $y + V_s(z) = R_s$ for $y \leq R_s$, which from equation 1 implies that $p(x, z)$ is constant on $x$ and $y$. 
Thus, the Law of One Price holds when the discount rate is zero. That is, if traders do not discount future payoffs, their expected value in participating in the market is the expected surplus from the future transaction, which varies by the same amount as their valuation. An increase in \( \epsilon > 0 \) in a buyer’s valuation implies an increase in \( \epsilon \) in their valuation from participating in the market, so \( x - V_b(x) \) is constant and prices are constant with regards to buyers and sellers valuations.

In the frictionless case, as the joint surplus from a meeting is constant

\[
[x - z - V_b(x) - V_s(z)] = R_b - R_s. \tag{6}
\]

Substituting this expression in equations 4 and 5 when \( r = 0 \) yields

\[
c_b = m(\theta)(1 - \omega) \max\{R_b - R_s, 0\} \tag{7}
\]

\[
c_s = m(\theta)\omega \max\{R_b - R_s, 0\}, \tag{8}
\]

and so \( R_b > R_s \). This means that among traders in the market, any buyer’s valuations is higher than any seller’s costs. Therefore, all meetings result in trade since there is no point in searching for other trade opportunities since they are all executed at the same price. In this case, the Law of One Price holds, and substituting equation 6 and \( y + V_s(z) = R_s \) into equation 1 we find the equilibrium price:

\[
\hat{p} = \omega R_b + (1 - \omega)R_s. \tag{9}
\]

If search costs \((c_b, c_s)\) converge to zero, then \( \hat{p} \) converges to the competitive price and \( R_s, R_b \) both converge to the same value \( R \), which is the competitive equilibrium price \( p^* \). In terms of quantity traded, the search equilibrium allocation also converges to \( sG(p^*) \) which is the quantity sold in competitive equilibrium.

As equations 4 and 5 are continuous on \( r \), if search costs \( c_b, c_s \) are strictly positive and \( r \) is lower than some threshold \( \hat{r} > 0 \), then all meetings result in trade. This implies that the steady-state equilibrium distribution of operating types \((\Phi, \Gamma)\) is given by the densities of \((F, G)\) on the types who participate \((v \geq R_b, c \leq R_s)\). For \( r \in (0, \hat{r}) \), all meetings result in trade and there is price dispersion (that is \( p(x, z) \) varies with \( x \) and \( y \)).

To solve for the equilibrium in this case note that, since all meetings result in trade, \( \max\{x - z - V_b(x) - V_s(z), 0\} = x - z - V_b(x) - V_s(z) \). If we substitute in equation 4 and differentiate with respect to \( x \), we have that \( V'_b(x) = (1 - \omega)m(\theta)/[r\theta + (1 - \omega)] \), which
is constant. Therefore, \( V_b(x) \) is linear and setting \( V'_b(R_b) = 0 \) yields the intercept. Using an analogous procedure, we can solve for \( V_s(z) \). Therefore, we have

\[
V_b = \frac{(1-\omega)m(\theta)}{(1-\omega)m(\theta) + r\theta}(x - R_b), \tag{10}
\]

\[
V_s = \frac{\omega m(\theta)}{\omega m(\theta) + r}(R_s - z). \tag{11}
\]

Substituting into equation 1 yields the equilibrium price for the transaction:

\[
p(x, z) = \omega \left[ r\theta x + \frac{(1-\omega)m(\theta)R_b}{r\theta + (1-\omega)m(\theta)} \right] + (1-\omega) \left[ \frac{ry + \omega m(\theta)R_s}{\omega m(\theta) + r} \right]. \tag{12}
\]

To solve for the set of equilibria where all meetings result in trade, we need to determine \( (\hat{r}, R_s, R_b) \). The market-clearing condition that \( R_s \) and \( R_b \) must satisfy is

\[
sG(R_s) = b[1 - F(R_b)]. \tag{13}
\]

Finally, to find \( R_s \) and \( R_b \), substitute equation 10 into 4, which gives

\[
\frac{c_b \theta}{m(\theta)} = (1-\omega) \int \left[ R_b - \frac{ry - \omega m(\theta)R_s}{r + \omega m(\theta)} \right] d\Gamma(z). \tag{14}
\]

In the steady-state when all meetings result in trade, the distributions of participating types are \( \Gamma(z) = G(z)/G(R_s) \) and \( \Phi(z) = F(x)/[1 - F(R_b)] \), and so

\[
\frac{c_b \theta}{m(\theta)} = (1-\omega) \int \left[ R_b - \frac{ry - \omega m(\theta)R_s}{r + \omega m(\theta)} \right] \frac{dG(z)}{G(R_s)}. \tag{15}
\]

Similarly, substituting equation 11 into 3 yields

\[
\frac{c_s}{m(\theta)} = \omega \int \left[ -R_s - \frac{r\theta x + (1-\omega)m(\theta)R_s}{r\theta + (1-\omega)m(\theta)} \right] \frac{dF(z)}{F(R_s)}. \tag{16}
\]

These two equations combined with the market-clearing condition determines \( (\theta, R_s, R_b) \). Mortensen and Wright (2002) show that there is a unique \( \hat{r} \) such that every meeting results in trade if and only if \( r < \hat{r} \). The condition that every meeting results in trade allows to keep the model easily tractable with positive search costs and price dispersion in equilibrium.
As shown in Mortensen and Wright (2002), as the discount rate $r$ and the search costs $(c_b, c_s)$ both converge to zero, then the search equilibrium prices all converge to $p^*$ and that the search equilibrium converges to the competitive equilibrium. Then, as it is the same allocation mechanism as the competitive equilibrium (as all trades occur at the competitive price) it is informationally efficient at this frictionless limit. We are interested in the allocations implemented by this mechanism away from the limit.

4.3 The Allocation Mechanism in the Search Equilibrium

Throughout our paper, we focus on the steady-state of the search mechanism since that is where the search mechanism involves the least information, providing a best-case scenario for the search mechanism. Note that in any steady-state, there is a constant distribution of types in the market. Therefore, the distribution of types leaving the market is the same as the distribution of types entering the market. These distributions are given by $(F, G)$ with the cutoffs $(R_b, R_s)$.

The allocation in the steady-state can be described by a pair $(p_s, y)$. The pricing function is $p_s : [0, 1] \to \mathbb{R}_+$, where $p_s(i)$ describes the equilibrium transaction price for agent $i$, if $i$ participates in the market. Recall that any buyer $i$ participates if $x^i \in [R_b, 1]$, and any seller $i$ participates if $y^i \in [0, R_s]$. If $i$ does not participate, then $p_s(i) = R_b$ for buyers or $p_s(i) = R_s$ for sellers.

Since buyers and sellers meet randomly and the transaction price depends on the pair of valuations of buyers and sellers $p(x, z)$, prices are not deterministic in the search equilibrium. However, the distribution of realized transaction prices is deterministic, as there is a continuum of traders. Prices can be summarized by a CDF $P : [p, \bar{p}] \to [0, 1]$. Any function $p_s$ consistent with the search equilibrium implies an equilibrium distribution of prices $P$. Let $\bar{x}(x)$ and $\bar{z}(z)$ be the highest seller’s cost and lower buyer’s valuation such that there is positive joint surplus in trading given buyer’s and seller’s valuations $(x, z)$, respectively. Then $p$ satisfies $p_s(i) \in \{p(x^i, y), y \in [y, \bar{y}(x^i)]\}$ if $i$ is a buyer and $p_s(i) \in \{p(x^i, y), x \in [\bar{x}(y^i), \bar{x}]\}$ if $i$ is a seller.

The privacy-preserving message process $(\mu_s, M_s)$ is constructed as follows:

The message space of the search mechanism is

$$M_s = \{(p_s, y) \in F \times Y : p_s(i)y_1(i) + y_2(i) = 0, \forall i\},$$

where $F$ is the space of functions on $[0, 1]$ to $\mathbb{R}_{++}$.  

16
Let $\mu^i_s$ be a correspondence from $E_i$ to $M_s$. Let $\mu^i_s : E^i \Rightarrow M_s$ given by

$$\mu^i_s(x^i) = \begin{cases} 
(1, -p_s(i)) & \text{if } i \in B \text{ and } x^i \geq p_s(i) \\
(0, 0) & \text{if } i \in B \text{ and } x^i < p_s(i) \text{ or } i \in S \text{ and } x^i > p_s(i) \\
(-1, p_s(i)) & \text{if } i \in S \text{ and } x^i > p_s(i)
\end{cases}.$$ 

Define the correspondence $\mu_s : E \Rightarrow M_s$ by

$$\mu_s(e) = \bigcap_i \mu^i_s(e^i) \cap (p_s(e) \times Y),$$  

where $p_s(e)$ is the pricing function determined by the search equilibrium in the environment $e$ (with buyers’ and sellers’ types distributed according to $F$ and $G$). The message correspondence $\mu_s$ is restricted to the subset of $M_s$ consistent with the search environment described in subsection 4.3. Note that $\mu_s$ is privacy-preserving by construction.

The search mechanism is defined by a triple $(\mu_s, M_s, g_s)$, where $g_s(p, y) = y$ is a projection from $M_s$ to $Y$. While the search equilibrium is a Nash equilibrium (everyone is best-responding), we do not explicitly require that the messages are part of a Nash equilibrium. See Maskin (1999) and Reichelstein and Reiter (1988) for the relevant complications related to realizing an allocation versus implementing one.

Note that $p_s(i) > R$ if $c^i \leq R$ and $p_s(i) < R$ if $v^i \geq R$ since prices must compensate for search costs, while traders who do not trade are the types with costs/valuations in $(R, R)$.

### 4.4 Informational Efficiency

For the search mechanism with positive frictions, the set of net trades is a higher dimensional object than the frictionless limit, which is the same as the competitive mechanism. To see this, consider that buyer of type $\phi \in \{1, \ldots, k\}$ can match with a seller of type $\gamma \in \{1, \ldots, k\}$. Consider a partition of the set of buyers of each type $\phi$ into $k$ subsets $\{m(\phi, \gamma)\}_{\gamma=1}^k$, corresponding to each seller type that a buyer could be matched with. Each subset $m(\phi, \gamma)$ can transact at a price $p(\phi, \gamma)$. If a pair of types $(\phi, \gamma)$ do not transact in the search equilibrium, then we can divide these into two cases: (1) a buyer of type $\phi$ does not participate in the market because his valuation is too low (so $\phi < R_b^k$, where $R_b^k$ is the reservation value of the marginal buyer type for the $k$-types economy). (2) A seller of type $s$ does not trade with the buyer because either $s$ does not participate or the discount rate.
$r$ is too high for all participating types to trade with each other. In case 1, the set $m(\phi, \gamma)$ is empty. In case 1, the buyer of type $\phi$ does not participate in the market, and we can just assume that $m(\phi, \gamma)$ has the same measure for each seller type $\nu$.

Let $\lambda(\phi, \gamma)$ be the probability that a transaction is between a buyer of valuation $x(\phi)$ and a seller of valuation $z(\gamma)$. For a buyer type $\phi$, who does participate in the market, $\lambda(\phi, \gamma)$ is given by the measure of the set $m(\phi, \gamma)$ divided by the sum of the measure of sets $\{m(b, \nu') : \nu' \in \{1, 2, \ldots, k\}\}$. If buyer type $\phi$ does not participate in the market, $\lambda(\phi, \gamma)$ is zero for all seller types.

The search message space in the $k$-environment specifies prices for each possible pairing of buyers and seller types, which means that there are $k^2$ prices for each pairing between the $k$-types of buyers and $k$-types of sellers. Let $y^B(\phi, \gamma) = (y^B(\phi, \gamma)_1, y^B(\phi, \gamma)_2)$ be the net trades of the set of buyers of type $\phi$ with sellers of type $\nu$ and $y^S(\phi, \gamma) = (y^S_1(\phi, \gamma), y^S_2(\phi, \gamma))$ be the net trades of the set of sellers of type $\nu$ with buyers of type $\phi$. The profile of net trades is $y = (y^B(\phi, \gamma), y^S(\nu, b)))_{\phi, \gamma \in \{1, \ldots, k\}}$.

Therefore, the set of net trades for the search mechanism is described by:

$$Y_k^s = \{y : I \to \mathbb{R}^2 : y(i) + \omega(i) \in X, \forall i \in I, \sum_{\phi, \gamma} \lambda(\phi, \gamma)[y^B(\phi, \gamma) + y^S(\phi, \gamma)] = (0, 0), y^B(\phi, \gamma) = y^S(\phi, \gamma) = 0 \text{ if } b < R_k\}.$$

(18)

Again, let $(\mu_s^k, M_s^k, g_s^k)$ be the $k$-environment versions of the search allocation mechanisms, where

$$M_s^k = \{(p, y) \in \mathbb{R}^{k^2}_{++} \times Y_k^s : p(\phi, \gamma)y^j(\phi, \gamma) + y^j_2(\phi, \gamma) = 0, \forall j \in \{B, S\}, \forall b, \nu \in \{1, \ldots, k\}\},$$

and $\mu_s^k$ is the finite analogue of $\mu_s$, a correspondence that maps $E^k$ into $M_s^k$, and $g_s^k$ is the projection from $M_s^k$ to $Y_k^s$.

For the search mechanism in the $k$-economy, there are $k^2$ prices and effectively $2k^2$ different types of traders (each player’s endowed type and who they are matched with) in the search equilibrium. Each buyer or seller must form expectations regarding prices for each of the $k$-types on the other side of the market. Therefore, each buyer or seller individually has to form expectations regarding $k$ distinct prices; each price depends on the opportunity costs, which depends on the distribution of types. This implies that the dimensional size of the message space is $k^2$ (prices) plus $\sum_{i=1}^k (k + k)$ different types of traders minus one dimension due to the market-clearing condition. Therefore, $M_s^k$ is a $3k^2 - 1$ dimensional manifold, which has approximately $1.5k$ times more dimensions than the message space of the competitive mechanism $M_c^k$. Lemma2 gives us the main
proposition regarding information size in the search economy.

**Lemma 2.** $M^k_m$ and $M^k_s$ are $2k$ and $3k^2 - 1$ dimensional manifolds.

**Proof.** See Appendix Subsection A.3

Combining Lemma 1 and 2, we can see that difference between the search and competitive mechanisms converges to infinity as $k \to \infty$. This is formalized in Proposition 1.

**Proposition 1.** As $k \to \infty$ the ratio of the dimensional size of $M^k_s$ to $M^k_c$ converges to infinity.

In other words, the search mechanism requires that each participant of the market be aware of all types of participants operating in the market to form expectations regarding payoffs from participating in the market and to bargain with the other participants. This is precisely the inverse of the intuition regarding the informational efficiency of the market as articulated by the literature on the informational efficiency of competitive markets: that each participant of the market can use prices as an efficient way to substitute for the information they would otherwise require to allocate resources without access to market prices.

## 5 The Informational Efficiency of Market-makers

### 5.1 Environment

Now suppose that, in addition to buyers and sellers (the traders), there is a finite set of market-makers in this economy, indexed by $j \in J$. Market-makers are firms that act as profit-maximizing intermediaries that "make the market" by posting bid and ask prices for the indivisible good, intermediating trade between the suppliers (sellers) and the final consumers (buyers) in the economy.\(^\text{12}\) Traders’ valuations are private information, so we assume market-makers are constrained to uniform pricing policies where there is no price discrimination.\(^\text{13}\) Unlike the search mechanism, buyers and sellers do not directly

\(^{12}\) In our model, market-makers perform the same role as in Spulber (1996a, 1996b), but in this model the number of market-makers is finite, and traders are matched with different probabilities to each market-maker.

\(^{13}\) A market-maker could practice price discrimination, but the trader’s valuations and awareness are private information. There is two-sided asymmetric information in the sense that both buyers and sellers have private information regarding their valuations and costs, as in Satterthwaite and Shneyerov (2007, 2008). Since the good is indivisible, the only direct revelation mechanism that is truthfully implementable consists of a pair of prices that the market-maker is willing to buy or sell the good for.
match with each other. Instead, both buyers and sellers trade through the market-makers. Buyers purchase from the lowest priced market-maker they have access too as long as it is lower than their valuation, while sellers sell at the highest-priced market-maker as long as the posted price is higher than their cost.

Consider the case of the absence of any form of frictions of trade: All traders have cost-less access to all contracts posted by all market-makers. We will show that this is equivalent to the competitive mechanism through Bertrand competition. To see this, consider a market-maker who posts a pair of bid and ask prices \((p_b, p_s)\), which are, respectively, higher and lower than the prices posted by all other market-makers. The market-makers profits are

\[
\pi(p_s, p_b) = (p_s - p_b) \times b[1 - F(p_s)],
\]

subject to the market clearing constraint that the quantity brought from sellers is equal to the quantity demanded by buyers:

\[
b(1 - F(p_s)) = sG(p_b).
\]

If the market-maker posts bid prices lower than some other market-maker, then no seller will sell to him or her, and its profits are zero. If the maker posts bid prices higher than all others but not the lowest ask prices, the market-maker has monopolized the supply and profits also satisfy 19 subject to the resource constraint 20.

**Proposition 2.** If at least two market-makers are operating, then there is only one Nash equilibrium: for at least two market-makers to post a pair of bid-ask prices \((p_b, p_s) = (p^*, p^*)\); market-makers post the competitive equilibrium price.

**Proof.** See Appendix Subsection A.4 □

Proposition 2 states that this environment of strategic price determination by market-makers implements the competitive equilibrium in a frictionless setting with complete awareness. Moreover, the proposition says that, without any frictions, two market-makers are sufficient to achieve the competitive allocation, as in a standard Bertrand model.14

We are interested in the case where there are imperfectly functioning markets and frictions exist. We develop such a model in the next few subsections. In particular, we as-

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14. Two is the magic number. In a strategic game with indivisibilities, Martinelli, Wang, and Zheng (2019) prove that all Nash equilibria with active trading are competitive if and only if there are at least two intra-marginal traders on each side of the market. They also show competition occurs in the lab with only two traders on each side.
sume that traders might not have full access to all market-makers, because traders might be simply unaware of the full set of market-makers operating in the market.

5.2 Frictions of Trading: Constrained Consideration Sets

The market-maker model incorporates frictions of trading, which allows it to yield results such as market-power, price dispersion, and other features of markets that do not exist in the competitive market mechanism. We model the frictions of trading by assuming that traders (buyers and sellers) have constrained consideration sets regarding the market-makers that they can trade with. Following Perla (2019) and Guthmann (2020), we use the term imperfect awareness to describe these constrained consideration sets regarding the market-makers that are operating.\footnote{Other studies, such as McAfee (1994), Perla (2019), Armstrong and Vickers (2020), and Albrecht (2020), use the terms "availability rate," "consideration set," "choice set," and "loyal," respectively, to indicate the subset of agents that buyers or sellers have access to and to indicate the degree of access among traders in the market.} We assume that awareness is randomly and independently distributed.

For $j \in J$ there is a measurable subset $A^j \subset [0, 1]$ of buyers and sellers who have access to market-maker $j$. Let $A^i = \{j \in J : i \in A^j\}$ be the subset of market-makers that trader $i$ has access to $j$. The set of environments $E$ includes the market-makers and the awareness among buyers and sellers regarding them (use the term "aware of $j$" to mean $j \in A^i$), and this information is given by $\{A^j\}_{j \in J}$. That is, $e^i = (x^i, A^i)$, where $x^i$ is trader $i$’s valuation or cost and $A^i$ is the set of market-makers that $i$ is aware of, an environment $e \in E$ is specified by $A = \{A^j\}_{j \in J}$, and the distributions of buyers valuations $F$ and sellers costs $G$.

Let the awareness parameter $m^j$ be given by $m^j = \lambda(A^j) \in (0, 1]$, where the (Lebesgue) measure of $A^j$ is the fraction of all traders aware of $j$. We assume that $A^j$ is a simple random sample of the traders. This means that it satisfies the properties of independence of type and awareness, which means that:

Assumption A1 The fraction of traders who are buyers in $A^j$ is $b$, and the fraction who are sellers is $s = 1 - b$.

Assumption A2 The distribution of types of buyers and sellers conditional to being in $A^j$ are $F$ and $G$, respectively.

Assumption A3 The probability of being aware of a competing market-maker is inde-
pendent, that is,
\[ \lambda(A^j \cap A^h) = \lambda(A^j) \times \lambda(A^h), \]
(21)
for the awareness sets \( A^j, A^h \) of marker-makers \( j \) and \( h \).

Note that equation 21 implies that the fraction of buyers and sellers who are aware of the seller \( j \) conditional on being aware of a competitor is \( m^j \). As valuations, traders’ consideration sets are private information; market-makers cannot price discriminate based on their consideration sets.

### 5.3 Static Market-maker Mechanism

The solution concept used here is Nash equilibrium in mixed strategies: a mixed strategy profile for the market-makers is a profile of distributions over a subset \( F \subset [\underline{\xi}, \bar{\xi}] \times [\underline{\xi}, \bar{\xi}] \) of pairs of prices that is consistent with market-clearing (that is, the quantity brought by the market-makers is equal to the quantity sold) such that any price pair on the support of the distributions is profit-maximizing. As stated in Proposition 3, given a profile of awareness parameters \( m \) such that there exists at most one market-maker of whom all traders are aware of, the unique equilibrium in this environment is a profile of pricing strategies described by \( \{P_j\}_{j=1}^J, p \) where \( P_j \) is a cumulative distribution function on \([0, 1]\) and \( p \) is a function that maps \([0, 1]\) into a pair of prices for buying and selling in \([\underline{v}, \bar{v}]\). Since buying and selling are mirror images of each other, we can think of a market-maker as choosing a point percentile \( \alpha \) on both the distribution of bid and ask prices. Figure 1 shows an example of two different \( P_j(\alpha) \) strategies.\(^{16}\) This choice is such that posting any pair of prices \( p(\alpha) = (p_s(\alpha), p_b(\alpha)) \) for \( \alpha \) on the support of \( P_j \) is profit-maximizing, and the resulting allocation is feasible.

We now have the relevant notation to characterize the equilibrium pricing. Figure 2 plots the equilibrium pricing strategies for two firms.

**Proposition 3 (Market-maker Equilibrium).** If \( m \) is such that \( m^j < 1 \) for at least \( J - 1 \) market-makers, then there is a unique equilibrium that consists of a profile of mixed pricing strategies \( \{P_j\}_{j \in J} \) and a sharing rule: for a pair market-makers \( h \) and \( g \), if \( m^h < m^g \), then traders aware of both will trade with \( h \) if the posted prices are the same.

The profile of equilibrium strategies features connected supports \([\underline{\alpha}^j, \bar{\alpha}^j] \) for each \( j \in J \), which share a common lower bound of the support \( \underline{\alpha} \). The distributions are continuous on \([\underline{\alpha}, 1)\).

\(^{16}\) With only two sellers, these pricing distributions are the same as in the differentiate goods model of Albrecht (2020).
each \( j \in J \), for \( \alpha \in [\bar{\alpha}^j, \bar{\alpha}^j] \), \( P^j(\alpha) \) satisfies

\[
P^j(\alpha) = \frac{m^j}{m^j} P^{\bar{j}}(\alpha),
\]

where \( \bar{j} \) is the market-maker with the largest awareness parameter \( m \). The distribution \( P^{\bar{j}} \) is given by

\[
\prod_{j \neq \bar{j}} (1 - m^j P^{\bar{j}}(\alpha)) Q(\alpha) = \prod_{j \neq \bar{j}} (1 - m^j) Q(1).
\]

where \( Q(\alpha) = [p_s(\alpha) - p_b(\alpha)] s G(p_b(\alpha)) \) is the quantity that is transacted per unit of awareness if there was no competition.

Proof. See Appendix Subsection A.5.

Figure 1: Example of equilibrium Price Distribution when there are two market makers 1, 2.

The equilibrium mixed strategy profile described in Proposition 3 is similar to the equilibrium described in McAfee (1994): the distributions of prices posted by the market-makers are non-degenerate and are continuous on the interior of the support, and the larger market-makers (in terms of the \( m^j \)) transact at higher margins than smaller market-makers in the sense that the distribution of margins between ask and bid prices of the larger market-makers first-order stochastically dominate those of the smaller market-makers. The reason for this result is that in an economy with finitely many market-makers it is less likely that buyers and sellers are aware of a competitor to a large market-maker.
than a competitor to a smaller market-maker, so the larger market-maker loses fewer customers if the spread between the buy and sell prices is increased.

After constructing the equilibrium strategies, it is easy to show they converge to the competitive equilibrium as the economy approaches full awareness of two market-makers.

**Corollary 1** (Convergence to Competitive Equilibrium). *Consider a sequence of awareness profiles $m_n$. If, for at least two market-makers $h, g$, $m_n^h$ and $m_n^g$ both converge to one, then the equilibrium pricing strategies $\{P^j, p\}_{j \in J}$ converge in probability to the competitive equilibrium price $p^*$, which is the unique equilibrium if $m^h = m^g = 1$ for at least two market-makers.*

### 5.4 Allocation Mechanism

The mechanism in this case implements the allocation corresponding to the Nash equilibrium in mixed strategies. Note that, if there is imperfect awareness regarding almost all the market-makers (that is, $m = (m^j)_{j=1}^J$ such that $m^j = 1$ for at most one $j$), then for any market-maker the posted price for buying is strictly smaller than for selling, and therefore profits are strictly positive.

Following Hurwicz (1977a), we interpret that the profits of the market-makers and the resulting deadweight losses are both components of the "cost" of operating the allocation mechanism: Hence, the allocation implemented by the mechanism features strictly nega-
tive net trades for the numeraire good among the traders in the economy. As bid and ask
prices diverge, not every buyer or seller who has access to a market-maker and would
trade under competitive prices does so.

The set of net trades incorporates the possibility of market-makers making profits by
buying at lower prices than they sell:

\[ Y_m = \left\{ y : I \rightarrow \mathbb{R}^2 : y(i) + \omega(i) \in X, \forall i, \int_0^1 y_1(i) di = 0, \int_0^1 y_2(i) di \leq 0 \right\} . \]

Given a realized profile of prices \( p_m = ((p_1^1, p_1^b), (p_2^1, p_2^b), \ldots, (p_J^1, p_J^b)) \), the message
space is given by

\[ M_m = \left\{ (p_m, y) \in \mathbb{R}^{2J}_{++} \times Y_m : \text{for each } i, \exists j \in A_i \text{ s.t. } i \in A_j \text{ and } p_j^i y_1(i) + y_2(i) = 0 \right\} , \]

and \( \mu_m \) is a correspondence on \( E \) to \( M_m \) that satisfies

\[ \mu_m = \cap_i \mu_m^i(e^i) , \]

where \( \mu_m^i : E^i \Rightarrow M_m \) satisfies

\[ \mu_m^i(e^i) = \left\{ (p_m, y) \in M_m : y_1(i) = \begin{cases} (0,0) & \text{if } i \in B \text{ and } v^i < \min\{p_j^i : j \in A^i\} \text{ or } A^i = \emptyset \\ (1, -\min\{p_j^i : j \in A^i\}) & \text{if } i \in B \text{ and } v^i \geq \min\{p_j^i : j \in A^i\} \\ (0,0) & \text{if } i \in S \text{ and } c^i > \min\{p_j^i : j \in A^i\} \text{ or } A^i = \emptyset \\ (-1, \min\{p_j^i : j \in A^i\}) & \text{if } i \in S \text{ and } c^i \leq \min\{p_j^i : j \in A^i\} \end{cases} \right\} . \]

5.5 Informational Efficiency

As in Subsection 4.4, consider a sequence of finite types economies \( \{e^k\} \) that approximates
the environment with the cumulative distributions of buyer and seller valuations \( F \) and \( G \).
In this case, the dimensional size of the message space incorporates the different market-
makers that make the market: If there are \( N \leq J \) market-makers with non-zero awareness
parameters, then there are \( 2N \) different prices posted to the traders, plus the subset of
traders who are not aware of any market-makers. As in the case of the search allocation
mechanism, the set of trader "types" increases to differentiate traders by their access to
different prices (as awareness is heterogeneous).

In this case the cardinality of the set of trader types \( I \) is determined by the discrete dis-
tributions of valuations \((G^k, P^k)\) and \(N\). The type of a trader can be specified by a triple \((r, \kappa, h)\), where \(r \in \{b, s\}\) denotes whether the trader is a buyer or seller, \(\kappa \in \{1, 2, \ldots, k\}\) denotes the valuation type of buyer or seller, and \(h \in J\) denotes the market-maker that the trader transacted with (including \(h = \emptyset\) if the trader does not transact with any market-maker). Therefore, there are \(2kN\) or \(2k(N + 1)\) types of traders if the subset of traders who are not aware of any market-makers is empty or non-empty, respectively. Therefore, market-clearing of the indivisible good among traders who interact with each market-maker implies that the message space corresponding to environments with \(k\) different valuations for buyers and sellers is a \(Z\)-dimensional manifold, where \(Z\) is equal to \(2N + (2k - 1)N\) or \(2N + (2k - 1)(N + 1)\), if the subset of traders who are not aware of any market-makers is empty or non-empty, respectively. This implies the following proposition:

**Proposition 4.** As \(k\) increases to infinity, the ratio of the dimensional size of the message spaces of the market-maker mechanism to the competitive mechanism \(Z/2k\) converges to \(N\) or \(N + 1\).

That is, the ratio of the size of the message spaces between the competitive mechanism and the market-maker mechanism is approximately the number of market-makers operating in the market. This result is intuitive since the competitive mechanism implicitly assumes a single monopolist market-maker called the Walrasian auctioneer whose bid and ask prices have zero spread.

However, for the model to generate more sophisticated market-behavior in equilibrium, such as price dispersion with average spreads between the ask and bid prices that depend on the tenure of the market-maker, we need to assume that at least two market-makers are operating. In that case, the minimum dimensional size of the message space of the market-maker equilibrium in a given period is \(Z = 4 + 2(2k - 1)\), as there is a pair of bid and ask prices and there are two profiles of net trades for the \(2k\) types of traders.

17. Note that the computation of the dimensional size of the message-space does not need to include the information of which other market-makers the trader was aware of besides the one he or she transacted with.
6 Dynamic Equilibrium: Competitive Price Formation and Awareness Diffusion

The previous section showed that the message space was proportional to the number of market-makers. In the best-case, static equilibrium, two market-makers could implement the competitive equilibrium through Bertrand competition. This section develops a dynamic model that shows how only a single strategic market-marker implements an equilibrium allocation that approximates the competitive equilibrium.

Consider the case of a monopolist market-maker who can deter the entry of other market-makers (we will explain how below). Assume the subset of traders who are not aware of any market-makers is empty. So, the number of trader types is $2^k$, but a pair of prices is realized instead of one price in the case of the Walrasian auctioneer. It represents the most informationally efficient mechanism in this class of market-maker environments with informational size $2^k + 1$, or only one dimension more than the competitive mechanism (outside of the limit case of $\delta = 0, \beta \to 1$, when it converges to perfect competition and there is only one price posted to all traders). This additional dimension reflects the profit margin between purchase and sale to provide incentives for the market-makers to "produce" the price mechanism.

To formally develop this model, suppose that time is discrete, $t = 0, 1, 2, \ldots$, and let $\beta = 1/(1 + r)$ be the discount factor. The good is perishable so in each period sellers can produce one unit of the good and buyers have unit demand per period.

**Awareness Diffusion:** Given a set $J$ of market-makers, there is an awareness profile $\{m^j_t\}_{j=1}^J \in (0, 1]^J$. Suppose awareness regarding a market-maker diffuses through the market according to

$$m^j_{t+1} = (1 - \delta)m^j_t + M(m^j_t, 1 - m^j_t), \quad (23)$$

where $M$ is a matching function that represents the diffusion of awareness through traders who hitherto had access to the market-maker, and $\delta \in [0, 1)$ is the awareness depreciation parameter (that is, the rate at which traders "forget" about the market-maker).

Each market-maker chooses in period zero to post prices according to a sequence of distributions for each period. Since the choice of the pricing strategies does not have any effect on the state of the market, the optimal strategy for each market-maker is to choose the profit-maximizing pricing behavior in each period given the action profile of the other market-makers in that period. This means that at a given point in time $t$, prices practiced in the market are given by $\{P^j_t\}_J$, described in the proof of Proposition 3.
We are interested in the convergence of equilibrium prices and allocation to the competitive equilibrium. Since the outcome of the equilibrium is stochastic as the market-makers randomize their bid and ask prices, we use the notion of convergence in probability: Convergence in probability of equilibrium prices and allocation to the competitive equilibrium means that the probability in a period \( t \) that prices and the allocation are at some positive distance from the prices and allocation of the competitive equilibrium converges to zero as \( t \to \infty \). The distance between two allocations \( y \) and \( y' \) that assign a consumption bundle \( y'(i) \) and \( y(i) \), respectively, to trader \( i \in [0,1] \) is described by a function \( D(y, y') \) that satisfies
\[
D(y, y') = \int_0^1 |y(i) - y'(i)|di.
\]

Proposition 5 follows from Proposition 3, as the expected equilibrium margin between buy and ask prices posted by the market-makers converges to zero if \( \lim m_j^t = 1 \) for \( m_j^t > 0 \) and \( J \geq 2 \). Therefore, the awareness of the traders regarding the market-makers operating converges to one as \( t \to \infty \) implies that a measure converging to one of the traders has access to buy and ask prices that are converging in probability to the competitive price. Therefore, the equilibrium allocation converges in probability to the competitive allocation.

**Proposition 5.** If the law of motion for awareness diffusion (equation 23) implies that \( \lim m_j^t = 1 \) for \( m_j^t > 0 \), then if \( J \geq 2 \) as \( t \to \infty \) the equilibrium prices and the equilibrium allocation converge in probability to the competitive equilibrium.

### 6.1 Steady-State Equilibrium

The analysis so far implied that a market where market-makers intermediate trade approximates the outcome of the competitive equilibrium as awareness regarding the market-makers diffuses. However, by allowing awareness to depreciate with \( \delta > 0 \) (as traders forget about a market-maker), given some assumptions on the matching function \( M^{18} \), there is a unique steady-state awareness level \( \hat{m} \) such that
\[
\hat{m} = M(\hat{m}, 1 - \hat{m}) / \delta.
\]

18. More precisely, that \( M \) is continuously differentiable, concave in both arguments, satisfies the condition that \( \lim_{a \to 0} \partial M(a, b) / \partial a = \infty, \forall b > 0 \).
There is a corresponding steady-state equilibrium if and only if all market-makers have the same awareness parameter $m^j = \hat{m}$, which is a symmetric mixed strategy described by a pair $\{P, p\}$ that all market-makers follow. If we let $\delta \to 0$, then awareness diffusion function $M$ implies that $\hat{m} \to 1$; that is, if awareness depreciates very slowly, the steady-state level of awareness approximates full awareness, which implies that the steady-state equilibrium approximates the competitive equilibrium.

**Entry and Exit**

Suppose that at some period $t$ some market-makers are operating in the market and some are not (which means that their awareness parameter $m^j$ is zero). There is an entry cost $E > 0$, which is the cost of setting up an entry-level awareness parameter $m_e \in (0, 1)$.

The law of motion for the diffusion of awareness (equation 23) implies that, with $M$ increasing and concave in both arguments, $m_e$ that is not very large, and if the depreciation parameter is not very large, then market-makers grow after entry (in the sense that $m^j_t$ is increasing over time). This implies that incumbent market-makers are larger than entrants and therefore transact at higher expected margins. If we follow the interpretation in Spulber (1996a) that market-makers are firms that intermediate between suppliers and consumers, this equilibrium property replicates the findings of Foster, Haltiwanger, and Syverson (2008, 2016) that incumbents charge higher prices than entrants.

### 6.2 Contestable Market Equilibrium

We will now show how a single market-maker ($N = 1$) approximates the competitive outcome if the market is contestable in the sense of Baumol (1982). Suppose that there are only two market-makers; that is, the set of market-makers is $J = \{1, 2\}$. Further suppose that $m^1_0 = 1, m^2_0 = 0$ at a date set to zero, so 1 is a monopolist market-maker and all traders have access to his posted contracts and 2 is out of the market. However, 2 can decide to enter in the current period. That is, 2’s action set is that she can choose $m^2_t \in \{0, m_e\}$ besides the price if she has not entered the market in the previous period. A monopoly deterrence equilibrium is a situation where the incumbent 1 chooses to post buying and selling prices for each period such that the profits from offering better prices to traders are too low to compensate for the cost of entering the market.

**Definition 3.** A **monopoly deterrence equilibrium** is an equilibrium where 1 chooses a pricing schedule and, given this pricing schedule, 2 finds it optimal to not enter. The pricing
schedule is profit-maximizing for two reasons. First, a higher selling–buying margin that yields higher profits for 1 would mean that 2 would enter and undercut 1’s posted offers in every period. Second, the schedule is profit-maximizing in the sense that it yields a higher discounted expected value of the profit stream for 1 than the expected value of the profits in the equilibrium under a duopoly if 2 also enters the market.

The proposition below states that, if entry costs are high enough and awareness diffusion is fast enough, then the unique equilibrium is for the monopolist to deter entry. This is because the entry cost is higher than the expected profits that can be obtained in the duopoly competition process where market-maker 2 competes with the former monopolist. However, monopolist 1 must commit to a sequence of prices that still yields a low enough profit to deter the entrant. The unique equilibrium is the sequence of prices that makes 2 indifferent between entering and not entering but that maximizes the present value of 1’s profit stream. As discount rates decrease, the present value of the gains from entering the market increase. This implies that the buy and ask prices posted by the monopolist become closer to the price of competitive equilibrium. As the discount rate $r$ converges to zero, the present value of any positive profit stream converges to infinity, which implies that the monopoly deterrence equilibrium converges to the competitive equilibrium as the discount rate converges to zero.

**Proposition 6.** If awareness diffusion is fast enough, such that $\sum_{t=0}^{\infty} (1 - m_{t}^{2}) \leq C$ for some constant $C$ conditional on market-maker 2’s entry, and the discount rate $r$ is low enough, then, for an entry cost $E$ equal or higher than $C \times \pi_{M}$, the unique equilibrium is the monopoly deterrence: The monopolist commits to post prices $p(\pi)$ that yield a per-period profit of

$$\pi = E / \left( \sum_{t=0}^{\infty} \beta^{t} m_{t}^{2} \right)$$

to deter entry. As $r$ converges to zero, the deterrence monopoly equilibrium profit flow $\pi$ converges to zero, which means the posted buying and selling prices converge to the competitive equilibrium prices $p^{*}$.

**Proof.** See Appendix Subsection A.6.

The existence of entry costs for 2 can be interpreted as the costs of communicating additional information to the market participants. If the costs of communicating additional
Proposition 6 shows that the equilibrium allocation converges to the competitive equilibrium allocation as the discount rate $r$ converges to zero. Therefore, even with a single active market-maker, when the discount rate is sufficiently low, competition "for the field"—to borrow a phrase from Demsetz (1968)—is sufficiently intense such that the equilibrium converges to the competitive equilibrium.

In the monopoly deterrence equilibrium, the number of market-making firms is $N = 1$, and because $m_0^1 = 1$, the subset of traders without access to a market-maker is empty. Therefore, Subsection 5.5 implies that the dimensional size of the message space of the allocation mechanism is $2 + (2k - 1)$ which is only one dimension more than the competitive allocation mechanism, which is $2k$. The added dimension is due to the fact that the market-maker posts a pair of prices, instead of only a single price. For completeness, we state it as a proposition.

**Proposition 7.** The message space of a monopoly deterrence equilibrium requires one more dimension than the competitive equilibrium.

Proposition 7 is stated in terms of the difference in dimension size, unlike previous propositions, which were in terms of the ratio. Given the competitive allocation is the unique informationally efficient (Jordan 1982), the mechanism of the monopoly deterrence market-maker is a second-best mechanism in terms of information. However, as we have argued above, the market-maker model has the added benefit of explicitly modeling a market microstructure with intermediators that facilitate trade.

### 7 Concluding Remarks

When evaluating the plausibility of different models, in addition to empirical relevance, we argue that an important factor to consider is the degree of informational efficiency. The size of the message space is a quick measure of the informational and computational burden placed on the agents in the model. In particular, we are interested in economies

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19. We have not performed a welfare analysis to check if the monopoly deterrence equilibrium is more efficient than the duopoly after 2’s entry. The social benefit of 2’s entry would be the reduction of the deadweight loss thanks to prices closer to perfect competition. The social benefits are different from the private benefits of entry.
with strategic agents where the allocation mechanism converges to the competitive mechanism. We study two such mechanisms: a search mechanism and a market-maker mechanism.

While the search model has been extensively studied in the literature, we show that it is unattractive from an informational perspective. In particular, we show that for large economies the search allocation mechanism requires infinitely more information than the competitive mechanism. A true search mechanism, where everyone must be able to search across all the people in the economy to find trading partners, is extremely inefficient in terms of information, as it requires that each agent must have a complete model of the economy. That is one possible reason we do not often observe single buyers trading with single sellers in real-world economies.

In contrast, we propose a different decentralized mechanism with market-makers. Such a mechanism has a few attractive features. First, the market-maker mechanism better matches certain features of the data, such as exhibiting price dispersion and prices that depend on the tenure of firms in the market. The other attractive feature, which is the focus of this chapter, is that the market-maker mechanism requires almost as little information as the competitive allocation, even when it is used to explain deviations from the competitive allocation. This informational efficiency is one possible reason we observe intermediaries that facilitate trade between individual original sellers and individual final buyers and that their operation is restricted to individual markets. It may be a puzzling result that an economy in which trading for each good is intermediated by market-makers can be thought of as more informationally efficient than markets where trading is highly decentralized, but it is an intuitive result: if traders only need to be aware of a few intermediaries for each good they purchase, the informational requirements are much smaller than if traders need to form a model of the whole market before engaging in search and bargaining for their consumption bundle.
A Appendix

A.1 Existence and Uniqueness of Walrasian Equilibrium Price

Proposition 8. There exists a unique $p^*$ such that $sG(p^*) = b[1 - F(p^*)]$.

Proof. Define the excess demand function as $Z : [0, 1] \to [-1, 1], Z(p) = s[1 - G(p)] - bF(p)$. At $p = 0$, $Z(p) = 1$, and at $p = 1$, $Z(p) = -1$. Since $F$ and $G$ are continuous, $Z$ is continuous. By the intermediate value theorem there exists a $p^*$ such that $Z(p^*) = 0$. Finally, from the fact that $Z$ is strictly decreasing, $p^*$ is unique. ■

A.2 Proof of Lemma 1

Proof. Using the conditions of $\sum_{i=1}^{k} y_i = 0$ and $py_1^i + y_2^i = 0, \forall i$, implies that the function $(p, y) \to (p, \tilde{y}) \in \mathbb{R}_{++} \times \mathbb{R}_{++}^{2k-1}$, where for $1 \leq i \leq 2k - 1, \tilde{y}^i = y^i$, is a $C^\infty$-diffeomorphism. Thus, $M_k^s$ is a $(2k - 1) + 1 = 2k$-dimensional manifold. ■

A.3 Proof of Lemma 2

Proof. The price vector of the search equilibrium has $k^2$ dimensions, while $Y_k^s$ has $2k^2$ dimensions. By analogous argument as for the competitive mechanism in Lemma 1, $M_k^s$ is a $k^2 + 2(k^2) - 1 = 3k^2 - 1$-dimensional manifold. ■

A.4 Proof of Proposition 2

Proof. To see that posting the competitive price is a Nash equilibrium, note that it yields zero profits. For any market-maker, a deviation either gives negative profits (if purchase prices are higher than $p^*$ and for selling are lower than $p^*$) or zero profits (in the case the purchase prices are lower than $p^*$ and for selling are higher than $p^*$). Therefore, there is no profitable deviation for a market-maker.

To see that this is the unique Nash equilibrium, note that, if market-makers post prices to make strictly positive profits, other market-makers could deviate and make profits by capturing the customers of competitor market-maker by posting more attractive bid and ask prices. This is the same as in a standard Bertrand model. ■
A.5 Proof of Proposition 3

Proof. Part 1. Existence and characterization:

As shown in Appendix A.1, there is a unique competitive equilibrium price, \( p^* \). Since \( A^j \) satisfies property 5.2, \( p^* \) is also the unique competitive equilibrium price for the subset of traders who are aware of a market-maker.

To construct the candidate equilibrium strategy profile \( \{ p^j \}_{j \in J} \), we consider pricing strategies described by a pair \((p_b, p_s)\) of offers to buy and sell the good by the market-maker where \( p_b \leq p^* \leq p_s \). First, consider the monopoly prices \( p^M = (p_b^M, p_s^M) \) which satisfies the monopolist market-maker problem:

\[
\max_{p_b, p_s} \left\{ (p_s - p_b) \min\{sG(p_b), b[1 - F(p_s)]\} \right\}. \tag{24}
\]

If there are multiple profit-maximizing pairs of monopoly prices, let \((p_b^M, p_s^M)\) be the pair of monopoly prices with the lowest difference between the buying and selling price that clears the market, \( sG(p_b^M) = b[1 - F(p_s^M)] \).

Let \( j \) be the market-maker with the largest awareness parameter: \( m^j = \max\{m^i\}_{i \in J} \). Let

\[
\alpha = \prod_{h \neq j} (1 - m^h),
\]

and let \( \Pi^M \) be the monopoly profit normalized in regards to \( m^j \in (0, 1] \)

\[
\Pi^M = (p_s^M - p_b^M)[sG(p_b^M)].
\]

Consider a function \( p : [0, 1] \to \mathbb{R}^2_+ \) such that \( p(\alpha) = (p_b(\alpha), p_s(\alpha)) \) is a pair of prices that satisfies

\[
[p_s(\alpha) - p_b(\alpha)]G[p_b(\alpha)] = \alpha \Pi^M / s, \tag{25}
\]

and satisfies market clearing,

\[
sG(p_b(\alpha)) = b[1 - F(p_s(\alpha))]. \tag{26}
\]

That is, \((p_b(\alpha), p_s(\alpha))\) is the pair of prices that implements a feasible net trade for a monopolist market-maker and yields a fraction \( \alpha \) of the monopoly profits. In addition, if for some \( \alpha \in [0, 1] \), there is more than one such pair of prices, then \((p_b(\alpha), p_s(\alpha))\) is the pair
with smallest difference between the buying and selling prices.

Formally, for each \( \alpha \in [0, 1] \), the prices \((p_b(\alpha), p_s(\alpha))\) satisfy

\[
(p_b(\alpha), p_s(\alpha)) = \arg\min_{(b, s)} \{|b - s| : (b, s) \text{ satisfies equations } 25, 26\}.
\]

To see that there exists at least one pair of prices that satisfies equations 25 and 26, note that, for \( p_b = p_s = p^* \), profits are zero and \( sG(p^*) = b[1 - F(p^*)] \). Profits for \( p_b = p^M_b \) and \( p_s = p^M_s \) are \( \Pi^M \), and they also satisfy market clearing. Since \( G \) and \( F \) are continuous, for any \( p_s \in [p^*, p^M_s] \), there exists a (unique) buying price \( p_b(p_s) \) that satisfies \( sG(p_s) = b[1 - F(p_b(p_s))] \). The continuity of \( G \) and \( F \) also imply that profits vary continuously from zero at \( p_s = p^* \) to \( \Pi^M \) at \( p_s = p^M_s \) and by the intermediate value theorem any profit level between zero and \( \Pi^M \) can be attained by some pair of prices \((p_s, p_b(p_s))\) with \( p_s \in [p^*, p^M_s] \).

Note also that, if a pair of prices is feasible for a monopolist market-maker, then competition between pairs of prices given by \( \{p(\alpha) : \alpha \in [0, 1]\} \) is also feasible. To see this, consider two market-makers in competition. Suppose each market-maker \( j \in 1, 2 \) posts a pair of bid and ask prices \( p(\alpha^j) \) for some \( \alpha^j \in [0, 1] \). If \( \alpha^1 < \alpha^2 \), 1 is posting lower ask prices and higher bid prices, and traders who are aware of both market-makers prefer to trade with 1. Since the traders’ preferences are independently distributed from trader awareness, the quantities supplied and demanded from 2 fall in the same proportion (both supply and demand from market-maker 2 decreases by a fraction of \( m^1 \)), so market-clearing still holds.

The candidate equilibrium strategy profile \( \{P_j\}_{j \in J} \) is a profile of CDFs on \( [\alpha, 1] \); \( P_j(\alpha) \) is the probability that buying (selling) prices higher (lower) than \( p_b(\alpha)(p_s(\alpha)) \) and that satisfies the equal profit condition

\[
\prod_{h \neq j}(1 - P_h(\alpha)m^h)[p_s(\alpha) - p_b(\alpha)]sG[p_b(\alpha)] = \alpha\Pi^M,
\]

where

\[
\frac{1 - m^h}{\text{Prob. } h \notin A^i} + \frac{[1 - P_h(\alpha)]m^h}{\text{Prob. } h \in A^i \text{ and } (p^h_b < p_b(\alpha) \text{ or } p^h_s > p_s(\alpha))} = 1 - P_h(\alpha)m^h,
\]

is the probability that a trader chooses to transact with the market-maker \( j \) over competitor \( h \) and \( \frac{\alpha}{\Pi^M} \) is the profit margin of a market-maker when posting prices at the minimum profitability level (lower bound for sales, upper bound for purchases), which
means its selling (buying) prices undercuts (tops) all competitors. Note that equation 27 implies that $P_h(\alpha) = 0$ as $[p_s(\alpha) - p_b(\alpha)]sG[p_b(\alpha)] = \alpha \Pi^M$.

To check that this is an equilibrium, note that any prices not in the support of equilibrium strategies $P = \{(p_b(\alpha), p_s(\alpha)) : \alpha \in [\bar{\alpha}, 1]\}$ lead to strictly lower profits: If we consider prices $(p_b, p_s) \in [p_b^M, p_b(\alpha)] \times [p_s(\alpha), p_s^M] \cap P$, they either yield strictly lower profits because they are undercut by prices which would achieve similar profitability in the case the market-maker were a monopolist or they are not feasible (that is, the market-maker promises to sell more than it purchases).

Given the sharing rule, it is easy to check that profits are constant on the support of $\{P_j\}$ for each $j$. If there is only one market-maker $j$ with the largest awareness parameter $m^\dagger$, then the equal profit condition (equation 27) implies that there is an atom of probability at $P_j(p^M)$, the monopoly price, in the mixed strategy of the largest market-maker. The sharing rule implies that traders always choose to trade with $h \neq j$ if $h$ posts the monopoly prices $p^M$, hence its profits do not fall discontinuously on the support of the equilibrium strategy $[\bar{\alpha}, 1]$ as $p(\alpha) \rightarrow p^M$.

Part 2. Uniqueness:

Note that any feasible pricing strategy for the market-maker is a pair of buying and selling prices that is consistent with market-clearing. Note that pricing strategies on the set of pairs of prices $\{p(a) : a \in [0, 1]\}$ are weakly dominant, as any feasible pricing strategy $(p_b, p_s)$ yields the same profits as a strategy $p(a)$ for some $a \in [0, 1]$, then sellers and buyers prefer the buying and selling prices $p(a)$. Therefore, for any market-maker posting a pair of prices $(p_b, p_s) \notin \{p(a) : a \in [0, 1]\}$ cannot be a best response to a best response. Hence, any candidate for Nash equilibrium consists of distributions over prices in $\{p(a) : a \in [0, 1]\}$.

As we are restricted in our candidate equilibrium strategies to prices in $\{p(a) : a \in [0, 1]\}$, then the proof of uniqueness of equilibrium is a proof of uniqueness over distributions on $[0, 1]$. Consider an equilibrium strategy profile $F$, undercutting arguments imply that $F = \{F_j\}_{j \in J}$ is non-degenerate and the upper bound of the support for at least a pair of market-makers must include the monopoly price (otherwise it is a profitable deviation to post the monopoly price). The union of the supports for the strategies must be convex; otherwise, market-makers could increase profits by posting prices in the complement of the support. Additionally, the supports for the mixed strategies of individual
market-makers must be convex; otherwise, the equal-profit condition will be violated.

Further, equal-profit conditions are required to hold in a mixed strategy equilibrium and these conditions imply that when the interior of the supports overlap, such that equation 27 holds. Assuming there are no atoms at the lower bound of the support of the distribution, the lower bound of the supports for any pair of seller types must be the same if the interiors of the supports overlap. This implies that equilibrium strategies for all seller types have convex supports (that is, the union of the supports is its own convex hull). Also, note that there cannot be atoms at a lower bound of the support of equilibrium price distributions; otherwise, other market-makers have the incentive to post a more attractive pair of prices $p(\alpha)$ for $\alpha$ in the $\epsilon$-neighborhood of the lower bound of the support.

It remains to show that any equilibrium profile of price distributions is such that the interiors of the supports overlap. That is, for any pair of market-makers the interior of the support of mixed pricing strategies must overlap.

To see that suppose, without loss of generality, that there is an equilibrium strategy profile $P$ such that there is a pair of market-makers $j, j'$ with the same awareness parameter $0 < m^j = m^{j'} < m^k$ who compete against each other posting prices according to a strategy that is described by pair of distributions $P^j, P^{j'}$ on $[0, 1]$ and the price posting function $p$ that maps $[0, 1]$ into pairs of buying and selling prices. The distributions $P^j, P^{j'}$ have the same support $[\alpha^*, \overline{\alpha}^*]$ while all other market-makers post prices according to distributions that have their supports in $[\alpha, 1]$ with $\alpha = \alpha^*$. In words, market-makers $j$ and $j'$ compete by posting strictly more attractive prices to buyers and sellers than all others. Let $\overline{K}$ be the market-maker with largest awareness parameter in the subset of market-makers $\hat{J} = J - \{j, j'\}$.

Let $\Pi^o(\alpha)$ be the equilibrium profits per unit of awareness of market-maker $o$ in posting prices $p(\alpha)$ we call that $o$’s equilibrium "profitability." Since $\overline{\alpha}^* = \overline{\alpha}^*$, the profitability of market-maker $j$ of posting prices $p(\overline{\alpha}^*)$ is

$$\Pi^j(\overline{\alpha}^*) / m^j = (1 - m^{j'}) \overline{\alpha}^* \Pi^M.$$  \hspace{1cm} (29)

If market-maker $o \neq j, j'$ posts prices $p(\overline{\alpha})$, its equilibrium profitability is

$$\Pi^o(\overline{\alpha}) = (1 - m^{j'}) (1 - m^j) \overline{\alpha} \Pi^M.$$  \hspace{1cm} (30)

Note that $\overline{\alpha} = \overline{\alpha}^*$, and therefore the equal-profit condition for $j$ and equation 29 imply
that

\[ \alpha^* = (1 - m^j) \alpha. \quad (31) \]

Finally, equations 30 and 31 together imply that for market-maker \( o \neq j, j' \) that its profitability in posting prices \( p(\alpha^*) \) satisfies

\[
\Pi^o(\alpha^*) = \alpha^* \Pi^M
\]

\[
= (1 - m^j) \alpha \Pi^M > (1 - m^j) (1 - m^{j'}) \alpha \Pi^M
\quad (33)
\]

\[
= \Pi^o(\alpha). \quad (34)
\]

The inequality 33 is a contradiction with \( P \) being an equilibrium. Therefore, in equilibrium, the interior of the supports must overlap.

Hence, the pricing strategy described by \( \{ P_j \}_{j \in J} \) and \( \{(p_b(\alpha), p_s(\alpha)) : \alpha \in [\alpha, 1] \} \) is the unique Nash equilibrium given the sharing rule that the market-maker with the smaller awareness parameter \( m^j \) has priority in transactions to buyers and sellers in the case of a tie in prices.

\[ \blacksquare \]

A.6 Proof of Proposition 6

Proof. As in the proof of proposition 3, let

\[ \pi^M = (p_s^M - p_b^M)sG(p_b^M) \]

be the monopoly profit rate. Consider a profit rate \( \pi \in (0, \pi^M) \), and suppose the incumbent 1 considers posting prices \( p(\pi) = (p_b(\pi), p_s(\pi)) \), which is the pair of bid and ask prices with the smallest difference that satisfies \([p_s(\pi) - p_b(\pi)]sG[p_b(\pi)] = \pi\). The pricing strategy, \( p(\pi) \), yields a payoff of \( \pi \times m^j \), where \( j \) is a monopolist. We will say a firm "undercuts" by posting a pair of bid and ask prices \( p(\hat{\pi}) \) with \( \hat{\pi} < \pi \), thus \( p_b(\hat{\pi}) > p_b(\pi) \) and \( p_s(\hat{\pi}) < p_s(\pi) \).

First, for simplicity, we consider the case where agents are myopic and only care about present payoffs. Then, we extend the equilibrium to the case when agents care about future payoffs.

Step 1: One period deterrence game
In this case, agents are myopic and only care about present payoffs so that the discount factor \( \beta = \frac{1}{1+r} = 0 \). In this case, the deterrence game has only one period. For \( \pi \leq E/m_e \), if the incumbent posts \( p(\pi) \), then the cost of entry \( E \) is higher than the profits 2 can make after entry by undercutting 1 with higher bid and lower ask prices. Therefore, 2 does not enter if 1 posts \( p \).

Therefore, if the incumbent posts \( p(\pi) \) for \( \pi = E/m_e \) (the highest profit margin that deters entry) and the entrant is playing "no entry," this is an equilibrium if 1 has no incentive to deviate. Clearly, 1’s profits decrease with lower bid-ask spread than \( \pi = E/m_e \), so there is no incentive for 1 to deviate by posting more attractive prices to the traders. While prices \( p(\pi') \) for \( \pi' > \pi = E/m_e \) imply that 2 can make profits higher than \( E/m_e \) if 2 enters and undercuts 1. Thus, 1 has no incentive to deviate in pure strategies.

It remains to show that 1 finds it more profitable to deter entry rather than to compete with 2 in mixed strategies, like described in Proposition 3. Note that the profit that 1 makes in the mixed strategy equilibrium with both market-makers operating is \( (1-m_e)\pi^M \). The profit 1 makes with entry deterrence strategy is \( E/m_e \), thus if the entry cost \( E \) is high enough so that

\[
E/m_e > (1-m_e)\pi^M
\]

(35)

the market-maker 1 finds it profitable to deter entry. Note also, that \( m_e(1-m_e)\pi^M \) are the profits of 2 if they enter and compete in mixed strategies with 1, thus if \( E > m_e(1-m_e)\pi^M \), 2 does not want to enter the market even if 1 plays the mixed strategy of the equilibrium where both market-makers compete. If \( E \leq m_e(1-m_e) \), 2 finds it a best response to enter the market and compete with 1 if 1 is playing the mixed strategy, and 1’s profits in the mixed strategy equilibrium are equal or higher than profit \( \pi = E/m_e \) of the deterrence strategy.

Therefore, the deterrence strategy \( p(\pi) \) for \( \pi = E/m_e \) for 1 and 2 chooses to not enter is the only equilibrium if and only if \( E > m_e(1-m_e)\pi^M \).

**Step 2: Infinite horizon deterrence game**

The logic of the one period case can be extended to the environment where agents are not myopic and instead have a common discount factor \( \beta \in (0,1) \). Then payoffs of 1 and 2 playing the mixed pricing strategies in the Markov perfect equilibrium after entry are
respectively

\[ U^1_e = \sum_{t=0}^{\infty} \beta^t (1 - m_t^2) \pi^M, \]  
\[ U^2_e = \sum_{t=0}^{\infty} \beta^t m_t^2 (1 - m_t^2) \pi^M, \]  
\[ \beta^t (1 - m_t^2) \pi^M, \]  
\[ \beta^t m_t^2 (1 - m_t^2) \pi^M, \]

where \( t \) is the number of periods after the entry. Therefore, \{\( m_t^2 \)\}_t is the sequence of awareness parameters for 2 that satisfies equation 23 for \( t > 0 \) and \( m_0^2 = m_e \).

Suppose the monopolist can only choose a fixed pricing schedule, posting \( p(\pi), \pi \in [0, \pi^M] \) in every period. The present value of 2’s profits conditional on entry when 1 is following its commitment \( p(\pi) \) is bounded above by

\[ U^2_d(\pi) = \sum_{t=0}^{\infty} \beta^t m_t^2 \pi. \]

To deter entry, \( \pi \) must imply that \( U^2_e(\pi) \leq E \). Consider the profit-maximizing strategy of entry deterrence \( \pi \) that satisfies \( U^2_e(\pi) = E \), substituting for A.6 and rearranging imply that \( \pi = E / (\sum_{t=0}^{\infty} \beta^t m_t^2) \). Thus, payoffs for the deterrence strategy for 1 are

\[ U^1_d = \frac{\pi}{(1 - \beta)} = \frac{E}{(1 - \beta) (\sum_{t=0}^{\infty} \beta^t m_t^2)} \]  
\[ (38) \]

In equilibrium with entry deterrence the monopolist must find deterring entry profitable: \( U^1_d \geq U^1_e \). Clearly, equation 38 implies that for an entry cost \( E \) high enough \( U^1_d > U^1_e \).

By assumption, \( m_t^2 \) converges to 1 at a fast enough rate so that

\[ \sum_{t=0}^{\infty} (1 - m_t^2) \leq C. \]

Now take a sequence \{\( \beta_n \)\} such that \( \lim \beta_n = 1 \). Let \( (U^1_d(\beta_n), U^2_d(\beta_n), U^1_e(\beta_n), U^2_e(\beta_n)) \) be the corresponding payoffs for 1 and 2 in deterrence and in the equilibrium with entry at the discount rate \( \beta_n \). Let \{\( \pi_n \)\}_n, with

\[ \pi_n = E / \left( \sum_{t=0}^{\infty} \beta_n^t m_t^2 \right) \]  
\[ (39) \]

for each \( n \), be the corresponding sequence of candidate deterrence equilibrium profit mar-
gins for the monopolist.

Set the entry cost $E \geq C \times \pi^M$. Then profits of 1 and 2 if both enter and play the mixed strategy equilibrium are bounded up by $C \pi M$, and so 2’s profits are always lower than the entry costs. Therefore, if 1 sets bid and ask prices $\pi_n$, 2 finds it optimal to not enter. Without entry, equation 38 implies that profits for 1, $U^1_d(\beta_n)$ are greater than $E$. Thus, if $E \geq C \times \pi^M$ the unique equilibrium is for 1 to deter entry, analogously to the one period case.

Note that $\sum (1 - m_2^t) \leq C$ and 39 imply that $\pi \to 0$ as $\beta \to 1$ and therefore $p(\pi)$ converges to $p_s = p_b = p^*$ as the discount rate $r$ falls to zero and the equilibrium allocation must converge to the competitive equilibrium.

Finally, relax the restriction that the monopolist is restricted to posting the same bid and ask prices for every period but chooses a sequence of bid and ask prices. Then the overall situation is similar but with added tedious notation. The monopolist chooses a sequence of profit shares $\{\pi_t\}_{t=0}^\infty$ with corresponding sequence of pairs of bid and ask prices $p(\pi_t)$. To deter entry the sequence $\{\pi_t\}$ must satisfy

$$\sum \beta^t m_2^t \pi_t \geq U^2_c,$$

the profits of the monopolist under this strategy are

$$U^1_d = \sum \beta^t \pi_t.$$

The profit-maximizing strategy for the monopolist is to choose, out of the sequences that satisfy the deterrence condition 40, the one that maximizes equation 41. Given that $m_2^t \to 1$ and is strictly increasing, there is a unique profit-maximizing sequence $\{\pi_t\}$, where the monopolist "frontloads" by extracting the highest profits in the early periods as the entrant’s profits from undercutting are relatively constrained by $m_2^t$ being smaller than in later periods from taking advantage of these higher margins. These profits are strictly higher than the profits from the strategy to commit to constant prices $(\pi/(1 - \beta))$, and therefore the previous arguments also apply in this case.

$\square$
References

Albrecht, Brian C. 2020. “Price Competition and the Use of Consumer Data.”


